Junior Balkan MO 2003

Kusadasi, Turkey

$\boxed{1}$ Let n be a positive integer. A number A consists of $2n$ digits, each of which is 4; and a number B consists of n digits, each of which is 8. Prove that $A + 2B + 4$ is a perfect square.

$\boxed{2}$ Suppose there are n points in a plane no three of which are collinear with the property that if we label these points as $A_1, A_2, \ldots, A_n$ in any way whatsoever, the broken line $A_1A_2 \ldots A_n$ does not intersect itself. Find the maximum value of n.

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$\boxed{3}$ Let D, E, F be the midpoints of the arcs BC, CA, AB on the circumcircle of a triangle ABC not containing the points A, B, C, respectively. Let the line DE meets BC and CA at G and H, and let M be the midpoint of the segment GH. Let the line FD meet BC and AB at K and J, and let N be the midpoint of the segment KJ.

a) Find the angles of triangle DMN;

b) Prove that if P is the point of intersection of the lines AD and EF, then the circumcenter of triangle DMN lies on the circumcircle of triangle PMN.

$\boxed{4}$ Let $x, y, z > -1$. Prove that

$$\frac{1+x^2}{1+y+z^2} + \frac{1+y^2}{1+z+x^2} + \frac{1+z^2}{1+x+y^2} \geq 2.$$

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